Supplemental note for Week 3 Part 1

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1 Vector products

For $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$

1.1 Dot product (=scalar)

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z = S$$

Equivalent Einstein convention: $S = A_\alpha B_\alpha$

1.2 Cross product (=vector)

$$\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y, A_x B_z - A_z B_x, A_x B_y - A_y B_x) = (V_x, V_y, V_z)$$

Equivalent Einstein convention: $V_\alpha = \epsilon_{\alpha,\beta,\gamma} A_\beta B_\gamma$, where $\epsilon_{\alpha,\beta,\gamma}$ is the Levi-Civita Symbol

$$\epsilon_{\alpha,\beta,\gamma} = \begin{cases} +1 & \text{if } (\alpha,\beta,\gamma) = (\mathrm{x,y,z}),\ (\mathrm{y,z,x}),\ \text{or } (\mathrm{z,x,y}) \\ -1 & \text{if } (\alpha,\beta,\gamma) = (\mathrm{z,y,x}),\ (\mathrm{y,x,z}),\ \text{or } (\mathrm{x,z,y}) \\ 0 & \text{if } \alpha = \beta,\ \alpha = \gamma,\ \text{or } \beta = \gamma \end{cases}$$

1.3 Dyadic product (=tensor)

$$\mathbf{AB} = \begin{pmatrix} A_x B_x & A_x B_y & A_x B_z \\ A_y B_x & A_y B_y & A_y B_z \\ A_z B_x & A_z B_y & A_z B_z \end{pmatrix} = \begin{pmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{pmatrix}$$

Equivalent Einstein convention: $M_{\alpha\beta} = A_\alpha B_\beta$