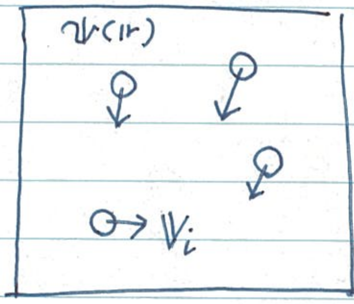
"The effects of hydrodynamics."

流体力学相互作用の取り扱い.

溶媒の速度場.



1). Colloidal Dispersions

$$\boxed{\mathbb{V}_n = \sum_m \mathbb{H}_{nm} \cdot \mathbb{F}_m}$$

i). Very Dilute Solution....

$$\mathbb{H}_{nm} = \frac{\mathbb{I}\,\delta_{nm}}{\zeta}$$

$$\longrightarrow V_n^\alpha = F_n^\alpha / \zeta$$ すべて独立.

ii) Oseen tensor

$$\mathbb{V}_n = v(\mathbb{R}_n) \quad , \quad g(\mathbf{r}) = \sum_n \mathbb{F}_n \delta(\mathbf{r} - \mathbb{R}_n)$$

$$\begin{cases} ① \quad \nabla \cdot v(\mathbf{r}) = 0 & \text{(連続の式.)} \\ ② \quad \eta \nabla^2 v(\mathbf{r}) + \nabla P = -g(\mathbf{r}) & \text{(Navier-Stokes 式.)} \end{cases}$$

フーリエ変換 $\quad v_{\mathbb{k}} \equiv \frac{1}{V} \int d\mathbf{r}\, v(\mathbf{r}) \exp(i\mathbb{k} \cdot \mathbf{r})$

$$\begin{cases} ③ \quad \mathbb{k} \cdot v_{\mathbb{k}} = 0 \\ ④ \quad -\eta k^2 v_{\mathbb{k}} - i\mathbb{k} P_{\mathbb{k}} = -g_{\mathbb{k}} \end{cases}$$

$\mathbb{k}\cdot$ ④ に ③を代入.

$$-\eta k^2 \mathbb{k}\cdot \mathbb{v}_{\mathbb{k}} - i k^2 P_{\mathbb{k}} = -\mathbb{k}\cdot \mathbb{g}_{\mathbb{k}}$$

⑤ $$P_{\mathbb{k}} = {}^{-i}/_{k^2}\ \mathbb{k}\cdot \mathbb{g}_{\mathbb{k}}$$

⑤を ④に代入.

$$-\eta k^2 \mathbb{v}_{\mathbb{k}} - i\mathbb{k}\frac{-i\mathbb{k}}{k^2}\cdot \mathbb{g}_{\mathbb{k}} = -\mathbb{g}_{\mathbb{k}}$$

$$\mathbb{v}_{\mathbb{k}} = \frac{1}{\eta k^2}\left(\mathbb{I} - \frac{\mathbb{k}\mathbb{k}}{k^2}\right)\cdot \mathbb{g}_{\mathbb{k}}$$

$$= \mathbb{H}_{\mathbb{k}}\cdot \mathbb{g}_{\mathbb{k}}.$$

$\downarrow$ 実空間に戻すと...

$$\mathbb{v}(\mathbb{r}) = \int d\mathbb{r}'\ \mathbb{H}(\mathbb{r}-\mathbb{r}')\cdot \mathbb{g}(\mathbb{r}')$$

$$\mathbb{H}(\mathbb{r}) = \frac{1}{8\pi\eta r}\left(\mathbb{I} + \frac{\mathbb{r}\mathbb{r}}{r^2}\right)$$

Oseen tensor //

$$
\begin{aligned}
\mathbb{V}_n &= \mathbb{v}(\mathbb{R}_n) \\
&= \int d\mathbb{r}' \, \mathbb{H}(\mathbb{R}_n - \mathbb{r}') \cdot \mathbb{g}(\mathbb{r}') \\
&= \int d\mathbb{r}' \, \mathbb{H}(\mathbb{R}_n - \mathbb{r}') \cdot \sum_m \mathbb{F}_m \, \delta(\mathbb{r}' - \mathbb{R}_m) \\
&= \sum_m \mathbb{H}(\mathbb{R}_n - \mathbb{R}_m) \cdot \mathbb{F}_m \\
&= \sum_m \mathbb{H}_{nm} \cdot \mathbb{F}_m
\end{aligned}
$$

Oseen $\underbrace{\text{tensor}}_{\text{そのまま}}$ では $\mathbb{H}_{nn}$ が発散してしまう.

$$
\therefore \begin{cases} \mathbb{H}_{nn} = \dfrac{\mathbb{I}}{\zeta} \\[2ex] \mathbb{H}_{nm} = \dfrac{1}{8\pi\eta r}\left(\mathbb{I} + \dfrac{\mathbb{r}\mathbb{r}}{r^2}\right) \qquad n \neq m \end{cases}
$$

とすることが多い.

Oseen tensor を使う方法の問題点.

i). 粒子の大きさを入れるのが困難.

ii). 粒子の距離が近い場合は使えない.

↳ まともに Navier-Stokes を解くことが必要.

$$\boxed{\mathbb{V}_n = \sum_m \mathbb{H}_{nm} \cdot \mathbb{F}_m}$$

Oseem tensor

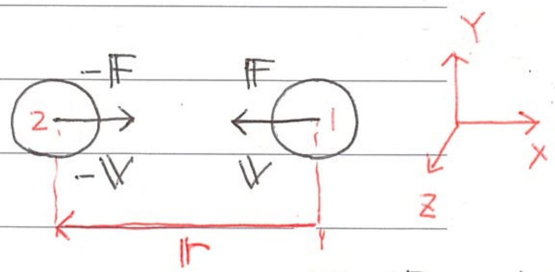
$$\begin{cases} \mathbb{H}_{nn} = \dfrac{\mathbb{I}}{6\pi\eta a} & \mathbb{r} = \mathbb{r}_m - \mathbb{r}_n \\ \mathbb{H}_{nm} = \dfrac{1}{8\pi\eta r}\left[\mathbb{I} + \dfrac{\mathbb{r}\mathbb{r}}{r^2}\right] & n \neq m \end{cases}$$

RPY tensor

$$\begin{cases} \mathbb{H}_{nn} = \dfrac{\mathbb{I}}{6\pi\eta a} \\ \mathbb{H}_{nm} = \dfrac{1}{8\pi\eta r}\left[\mathbb{I} + \dfrac{\mathbb{r}\mathbb{r}}{r^2} + \dfrac{2}{3}\left(\dfrac{a}{r}\right)^2\left(\mathbb{I} - \dfrac{3\mathbb{r}\mathbb{r}}{r^2}\right)\right] \end{cases}$$

Quiz

i) squeeze



$$\begin{cases} \mathbb{V}_1 = \mathbb{H}_{11} \cdot \mathbb{F}_1 + \mathbb{H}_{12} \cdot \mathbb{F}_2 \\ \mathbb{V}_2 = \mathbb{H}_{22} \cdot \mathbb{F}_2 + \mathbb{H}_{21} \cdot \mathbb{F}_1 \end{cases}$$

$\mathbb{F} = (F, 0, 0)$
$\mathbb{V} = (V, 0, 0)$
$\mathbb{r} = (r, 0, 0)$

Oseem

$$V = \frac{1}{6\pi\eta a} F + \frac{1}{8\pi\eta r}[1 + 1](-F)$$

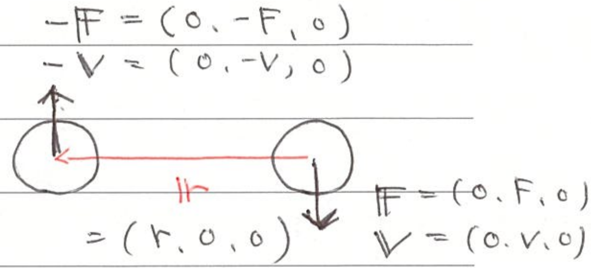
$$= \left[\frac{1}{6\pi\eta a} - \frac{1}{4\pi\eta r}\right] F$$

$$\frac{V}{F} = \frac{1}{6\pi\eta a} - \frac{1}{4\pi\eta r}$$

RPY

$$\frac{V}{F} = \frac{1}{6\pi\eta a} - \frac{1}{4\pi\eta r} + \frac{1}{6\pi\eta r}\left(\frac{a}{r}\right)^2$$

ii) tangential.



Oseen

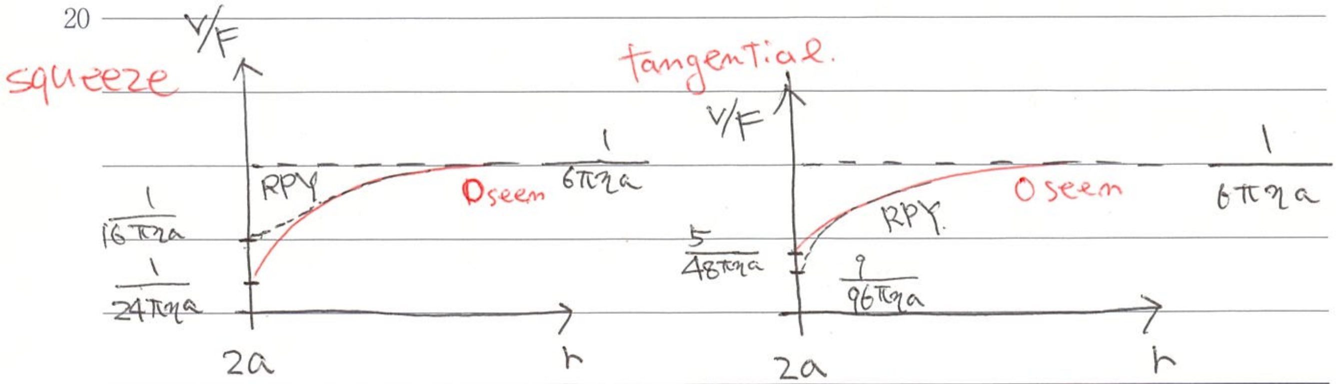
$$V = \frac{1}{6\pi\eta a} F + \frac{1}{8\pi\eta r}[1 + 0](-F)$$

$$= \left[\frac{1}{6\pi\eta a} - \frac{1}{8\pi\eta r}\right] F$$

$$\frac{V}{F} = \frac{1}{6\pi\eta a} - \frac{1}{8\pi\eta r}$$

RPY

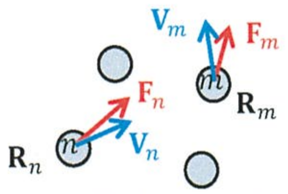
$$\frac{V}{F} = \frac{1}{6\pi\eta a} - \frac{1}{8\pi\eta r} - \frac{1}{12\pi\eta r}\left(\frac{a}{r}\right)^2$$



3. Summary for hydrodynamic interactions (HI) between particles

Ryoichi Yamamoto

1. Mobility tensor



Suppose that a collection of N spherical particles, all having the same radius a, are suspended in an incompressible fluid with the viscosity η. Let $\mathbf{R}_1, \mathbf{R}_2, \cdots, \mathbf{R}_N$ be the positions of the particles and $\mathbf{F}_1, \mathbf{F}_2, \cdots, \mathbf{F}_N$ be the force acting on them. We assume that there are no external torques acting on the particles and inertia effects are all neglected due to very small Re. Then the velocities of the particles are written as

$$\mathbf{V}_n = \frac{d\mathbf{R}_n}{dt} = \sum_{m=1}^{N} \mathbf{H}_{nm} \cdot \mathbf{F}_m$$

by using the mobility tensor $\mathbf{H}_{nm}$. Three representations of $\mathbf{H}_{nm}$ with different levels of approximations are summarized below.

I) No HI:

$$\mathbf{H}_{nn} = \frac{1}{6\pi\eta a}\mathbf{I}$$
$$\mathbf{H}_{nm} = 0 \qquad (n \neq m)$$

II) Oseen tensor:

$$\mathbf{H}_{nn} = \frac{1}{6\pi\eta a}\mathbf{I}$$
$$\mathbf{H}_{nm} = \frac{1}{8\pi\eta r}\left[\mathbf{I} + \frac{\mathbf{rr}}{r^2}\right] \qquad (n \neq m)$$

III) Rotne-Prager-Yamakawa (RPY) tensor:

$$\mathbf{H}_{nn} = \frac{1}{6\pi\eta a}\mathbf{I}$$
$$\mathbf{H}_{nm} = \frac{1}{8\pi\eta r}\left[\mathbf{I} + \frac{\mathbf{rr}}{r^2} + \frac{2}{3}\left(\frac{a}{r}\right)^2\left(\mathbf{I} - \frac{3\mathbf{rr}}{r^2}\right)\right] \qquad (n \neq m)$$

Here,

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{r} = \mathbf{R}_n - \mathbf{R}_m, \quad r = |\mathbf{r}|$$