## edX

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## Homework 3

## Homework 3-1

0.0/1.0 point (graded)

Calculate the auto-correlation function $\varphi_{Y}(t)$ for a dynamic process $Y(t)=A \sin \left(\omega_{1} t+\pi\right)$. Choose the correct answer from the following choices.
$\varphi_{Y}(t)=\frac{A^{2}}{2} \cos \left(\omega_{1} t\right)$
$\varphi_{Y}(t)=\frac{A^{2}}{2} \sin \left(\omega_{1} t\right)$
$\varphi_{Y}(t)=\frac{A^{2}}{2} \cos \left(\omega_{1} t+\pi\right)$$\varphi_{Y}(t)=\frac{A^{2}}{2} \sin \left(\omega_{1} t+\pi\right)$

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## Homework 3-2

0.0/1.0 point (graded)

Calculate the auto-correlation function $\varphi_{Y}(t)$ for a dynamic process $Y(t)=A \cos \left(\omega_{1} t\right)+\boldsymbol{B} \xi(t)$, where $\xi(t)$ is the White noise introduced in Part 1. Choose the correct answer from the following choices.

$$
\varphi_{Y}(t)=\frac{A^{2}}{2} \cos \left(\omega_{1} t\right)+A B \sqrt{\cos \left(\omega_{1} t\right) \delta(t)}+B^{2} \delta(t)
$$

$$
\varphi_{Y}(t)=\frac{A^{2}}{2} \cos \left(\omega_{1} t\right)+A B \sqrt{2 \cos \left(\omega_{1} t\right) \delta(t)}+B^{2} \delta(t)
$$

$\varphi_{Y}(t)=\frac{A^{2} B^{2}}{2} \cos \left(\omega_{1} t\right) \delta(t)$
$\varphi_{Y}(t)=\frac{A^{2}}{2} \cos \left(\omega_{1} t\right)+B^{2} \delta(t)$$\varphi_{Y}(t)=\frac{A^{2}}{2} \cos \left(\omega_{1} t\right)-B^{2} \delta(t)$

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## Homework 3-3

0.0/1.0 point (graded)

Estimate the diffusion constant $D$ of spherical particles with radius $a=1 \mu \mathrm{~m}$ immersed in water at $T=300 \mathrm{~K}$ using the Stokes-Einstein relation (Eq.(32))

$$
\begin{equation*}
D=\frac{k_{B} T}{6 \pi a \eta} \tag{32}
\end{equation*}
$$

and the following parameters

- Viscosity of water at room temperature: $\eta=0.85 \times 10^{-3} \mathrm{~Pa} \cdot \mathrm{~s}$
- The Boltzmann constant: $k \quad{ }_{B}=1.380649 \times 10 \quad{ }^{-23} \mathrm{~J} \cdot \mathrm{~K}^{-1}$

Choose the value closest to your answer from the following choices.

| $2.6 \times 10^{-7} \mathrm{~m}^{2} \cdot \mathrm{~s}^{-1}$ |  |
| :--- | :--- |
| $2.6 \times 10^{-13} \mathrm{~m}^{2} \cdot \mathrm{~s}^{-1}$ |  |
|  | $2.6 \times 10^{-19} \mathrm{~m}^{2} \cdot \mathrm{~s}^{-1}$ |
| $2.6 \times 10^{-7} \mathrm{~m}^{-2} \cdot \mathrm{~s}$ |  |
| $2.6 \times 10^{-13} \mathrm{~m}^{-2} \cdot \mathrm{~s}$ |  |
| $2.6 \times 10^{-19} \mathrm{~m}^{-2} \cdot \mathrm{~s}$ |  |

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## Homework 3-4

0.0/1.0 point (graded)

Calculate the right-hand-side of Eq.(47) using the correlation function given in Eq.(26).

$$
\begin{align*}
\varphi_{V}(t) & =\frac{3 \tilde{D}}{\zeta m} \exp \left(-\frac{\zeta}{m}|t|\right)  \tag{26}\\
D & =\frac{1}{3} \int_{0}^{\infty} \mathrm{d} t \varphi_{V}(t) \tag{47}
\end{align*}
$$

Choose the correct result for $D$ from the following choices.
$6 \tilde{D} t$
$\frac{\tilde{D}}{k_{B} T \zeta}$
$\frac{m \tilde{D}}{\zeta}$
$\frac{\tilde{D}}{\zeta^{2}}$

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## Homework 3-5

0.0/2.0 points (graded)

Replace $F_{0} \rightarrow 2 F_{0}$ in Eq.(41), then redo all the calculations to derive the equation corresponding to equation Eq.(47). Choose the correct equation, relating the diffusion constant $D$ to the velocity auto-correlation funciton $\varphi_{V}(t)$, from the following choices.$D=\frac{2}{3} \int_{0}^{\infty} \mathrm{d} t \varphi_{V}(t)$$D=\frac{\sqrt{2}}{3} \int_{0}^{\infty} \mathrm{d} t \varphi_{V}(t)$$D=\frac{1}{3} \int_{0}^{\infty} \mathrm{d} t \varphi_{V}(t)$$D=\frac{1}{3 \sqrt{2}} \int_{0}^{\infty} \mathrm{d} t \varphi_{V}(t)$$D=\frac{1}{6} \int_{0}^{\infty} \mathrm{d} t \varphi_{V}(t)$

## Submit You have used 0 of 2 attempts

