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Homework 3

Homework 3-1

0.0/1.0 point (graded)

Calculate the auto-correlation function $\varphi_Y(t)$ for a dynamic process $Y(t) = A \sin(\omega_1 t + \pi)$. Choose the correct answer from the following choices.

$$\bigcirc \varphi_Y(t) = \frac{A^2}{2} \cos(\omega_1 t)$$

$$\bigcirc \varphi_Y(t) = \frac{A^2}{2} \sin(\omega_1 t)$$

$$\bigcirc \varphi_Y(t) = \frac{A^2}{2} \cos(\omega_1 t + \pi)$$

$$\bigcirc \varphi_Y(t) = \frac{A^2}{2} \sin(\omega_1 t + \pi)$$

Submit You have used 0 of 2 attempts

Homework 3-2

0.0/1.0 point (graded)

Calculate the auto-correlation function $\varphi_Y(t)$ for a dynamic process $Y(t) = A \cos(\omega_1 t) + B\xi(t)$, where $\xi(t)$ is the White noise introduced in Part 1. Choose the correct answer from the following choices.

$$\bigcirc \varphi_Y(t) = \frac{A^2}{2} \cos(\omega_1 t) + AB\sqrt{\cos(\omega_1 t)\,\delta(t)} + B^2\delta(t)$$
$$\bigcirc \varphi_Y(t) = \frac{A^2}{2} \cos(\omega_1 t) + AB\sqrt{2\cos(\omega_1 t)\,\delta(t)} + B^2\delta(t)$$
$$\bigcirc \varphi_Y(t) = \frac{A^2B^2}{2} \cos(\omega_1 t)\,\delta(t)$$
$$\bigcirc \varphi_Y(t) = \frac{A^2}{2} \cos(\omega_1 t) + B^2\delta(t)$$

$$\bigcirc \varphi_Y(t) = \frac{A^2}{2} \cos(\omega_1 t) - B^2 \delta(t)$$

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You have used 0 of 2 attempts

Homework 3-3

0.0/1.0 point (graded)

Estimate the diffusion constant D of spherical particles with radius $a = 1 \mu m$ immersed in water at T = 300K using the Stokes-Einstein relation (Eq.(32))

$$D = \frac{k_B T}{6\pi a\eta} \tag{32}$$

and the following parameters

- Viscosity of water at room temperature: $\eta = 0.85 \times 10^{-3}$ Pa·s
- The Boltzmann constant: $k_{B} = 1.380649 \times 10^{-23} \text{J} \cdot \text{K}^{-1}$

Choose the value closest to your answer from the following choices.

$$\begin{array}{|c|c|c|c|c|} \hline & 2.6 \times 10^{-7} \text{m}^2 \cdot \text{s}^{-1} \\ \hline & 2.6 \times 10^{-13} \text{m}^2 \cdot \text{s}^{-1} \\ \hline & 2.6 \times 10^{-19} \text{m}^2 \cdot \text{s}^{-1} \\ \hline & 2.6 \times 10^{-7} \text{m}^{-2} \cdot \text{s} \\ \hline & 2.6 \times 10^{-13} \text{m}^{-2} \cdot \text{s} \\ \hline & 2.6 \times 10^{-19} \text{m}^{-2} \cdot \text{s} \end{array}$$

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You have used 0 of 2 attempts

Homework 3-4

0.0/1.0 point (graded)

Calculate the right-hand-side of Eq.(47) using the correlation function given in Eq.(26).

$$\varphi_V(t) = \frac{3\tilde{D}}{\zeta m} \exp\left(-\frac{\zeta}{m}|t|\right)$$
 (26)

$$D = \frac{1}{3} \int_0^\infty \mathrm{d}t \; \varphi_V(t) \tag{47}$$

Choose the correct result for D from the following choices.



Homework 3-5

0.0/2.0 points (graded)

Replace $F_0 \rightarrow 2F_0$ in Eq.(41), then redo all the calculations to derive the equation corresponding to equation Eq.(47). Choose the correct equation, relating the diffusion constant D to the velocity auto-correlation function $\varphi_V(t)$, from the following choices.

$$O D = \frac{2}{3} \int_0^\infty dt \, \varphi_V (t)$$

$$D = \frac{\sqrt{2}}{3} \int_0^\infty dt \, \varphi_V (t)$$

$$D = \frac{1}{3} \int_0^\infty dt \, \varphi_V (t)$$

$$D = \frac{1}{3\sqrt{2}} \int_0^\infty dt \, \varphi_V (t)$$

$$D = \frac{1}{6} \int_0^\infty dt \, \varphi_V (t)$$

