# 自走する粒子系としての細胞や生物集団のふるまい <br> Collective behaviors of cells and lives modelled as self－propelled particles 

Ryoichi Yamamoto

## 70 unsolved problems in Physics（JPS 2017）

## 日本物理学会創立 70 周年記念企画 ？ <br> 物理学70の不思議



## \＃63：Collective behaviors of cells and lives modelled as self－propelled particles

63 自走する粒子系としての細胞や生物集団のふるまい

混雑した駅の通路や交差点を歩いているうちに，行き交 う人の流れに自然と「レーン」が形成されていることに気 づくことがある。こうしたヒトの群集に限らず，動物の群 れや微生物の集団を観察してみると，何かに命令されるこ となく，運動の様子や配置•配列に，ある種のパターンや構造が自発的に現れる例がいくつも見つかる（図左：バク テリア（枯草菌）が集団運動している様子）
互いに影響し合いながら運動する同種の粒子や要素が呈 する巨視的なふるまいを知り，その背後にある数理的な共通性や差異を明らかにするのは，統計物理の重要なテーマ の 1 つである。細胞や動物，人工的な移動物体である自動車など，自ら運動する粒子や物体を「同種の粒子」とみな し，自己駆動粒子系やアクティブマターとよんで，その多体的•統計的なふるまいを明らかにしようという試みが広 がっている。自己駆動粒子系は複雑でその例は多岐にわた るため，それらをすっきりと分類することは難しい。一般 に，粒子自体が異方的で，エネルギー散逸と注入をともな い，粒子間の実効的な相互作用に作用•反作用の法則が成 り立たない。また，局所的なゆらぎが緩和せず系全体にお よび，疎密や対流，渦などが自己組織される（図右：竜巻


状の鳥（ムクドリ）の渦巻き）などの点において，アクティ ブではない粒子の系とはふるまいが大きく異なる
ゆらぎをともないながら自走する粒子を単純化した数理 モデルとして，ビチェックモデル（Vicsek model）がよく調 べられている。周囲と進行方向をそろえようとする効果と ゆらぎとの競合によって，運動方向の秩序が不連続的に相転移することが知られている。はじめはランダムに歩行し ていた昆虫の集団が，ある密度を超えると 1 方向に行進を はじめる現象などが，このモデルでよく説明できるという報告もある。こうした研究によって，生物や人工物の「群 れ」の数理的な理解が進みつつある

## Major categories of self-propelled particles

## Dry particles

in which hydrodynamics is not important ex. flock of birds

Wet particles
in which hydrodynamics plays crucial role
ex. school of fish


## A popular dry particles: Active Brownian particles

- MIPS: Cates, Tailleur, Ann. Rev. Cond. Matt. 6, 219 (2015)
- Omar, Klymko, GrandPre, Geissler, PRL 126, 188002 (2021)

$$
\dot{\mathbf{r}}=v_{0} \hat{\mathbf{n}}-(m \xi)^{-1} \nabla V(\mathbf{r})+\sqrt{2 D} \boldsymbol{\eta}_{\text {trans }}(t)
$$

$$
\text { time }=0.0 \tau
$$

$$
\begin{aligned}
& \hat{\mathbf{n}}=(\cos \theta, \sin \theta) \\
& \dot{\theta}=\sqrt{2 D_{r}} \eta_{\mathrm{rot}}(t) .
\end{aligned}
$$



$$
\phi=0.5
$$



## Another popular dry particles: Vicsek model

- Tamás Vicsek et al., PRL 75, 1226 (1986)


$$
\theta_{j}^{t+1}=\arg \sum_{k \sim j} \mathrm{e}^{\mathrm{i} \theta_{k}^{t}}+\eta_{j}^{t}
$$

$$
\boldsymbol{r}_{j}^{t+1}=\boldsymbol{r}_{j}^{t}+v_{0} \boldsymbol{e}_{\theta_{j}^{t+1}}
$$

$$
\begin{aligned}
& \partial_{t} \boldsymbol{v}+\lambda_{1}(\boldsymbol{v} \cdot \boldsymbol{\nabla}) \boldsymbol{v}+\lambda_{2}(\boldsymbol{\nabla} \cdot \boldsymbol{v}) \boldsymbol{v}+\lambda_{3} \boldsymbol{\nabla}\left(|\boldsymbol{v}|^{2}\right) \\
& =\alpha \boldsymbol{v}-\beta|\boldsymbol{v}|^{2} \boldsymbol{v}-\boldsymbol{\nabla} P+D_{B} \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{v}) \\
& \quad \quad+D_{T} \boldsymbol{\nabla}^{2} \boldsymbol{v}+D_{2}(\boldsymbol{v} \cdot \boldsymbol{\nabla})^{2} \boldsymbol{v}+\boldsymbol{f}, \\
& P=P(\rho)=\sum_{n=1}^{\infty} \sigma_{n}\left(\rho-\rho_{0}\right)^{n}, \\
& \frac{\partial \rho}{\partial t}+\boldsymbol{\nabla} \cdot(\boldsymbol{v} \rho)=0 .
\end{aligned}
$$

Vicsek model (discrete) $\rightarrow$ Toner-Tu model (continuum)

Fluid \& Soft Matter UvA on YouTube

## Other dry particles

There existed a few good models before Vicsek.

## Dry particles for CG: Boids

- artificial life program proposed by Craig Reynolds (1986)


2) Alignment
3) Cohesion


## Dry particles in Japanese article：Sakai model

－Sumiko Sakai，生物物理 13， 83 （1973）

A B－1

（A）個体 j の自己運動．（B）接近運動 B－2は，引力Cと個体間の距離 $R \neq$ との関係を示す。（C）整列運動。

$$
\begin{aligned}
& m \frac{d^{2} \mathbf{X}}{d t^{2}}+\nu \frac{d \mathbf{X}}{d t}=\mathbf{F} \\
& \mathbf{F}_{a j}=a\left(\frac{d \mathbf{X}_{j}}{d t}\right) \backslash\left|\frac{d \mathbf{X}_{j}}{d t}\right| \\
& \mathbf{F}_{a j}=\frac{1}{N-1} \sum_{i=1}^{N} \frac{C\left(R_{j i}\right)\left(\mathbf{X}_{i}-\mathbf{X}_{j}\right)}{R_{j i}}, \\
& \mathbf{F}_{h j}=\frac{1}{M} \sum_{\mathbf{x} i \in A j} h\left(\frac{d \mathbf{X}_{i}}{d t}-\frac{d \mathbf{X}_{j}}{d t}\right) \\
& \mathbf{F}_{b}=\mathbf{b}(t)
\end{aligned}
$$

（2）propulsion
（3）attraction $\quad+$ collision
（4）aligning
（5）random

## Dry particles in Japanese article：Sakai model

## A




図3 群れのパターンの種類
（i）アメーバ状運動
推進力が小さく擾乱が大きいと，群れは小さくかた まりほとんど止っている。形はほほ円形であるが，周辺の形状は刻々様々に変化する（図 $3-\mathrm{A}$ ）。
（ii）ドーナツ形運動
推進力が大きくなると互いに輪状になって回転し，中心部に空洞のある群れになる。群れの重心はほとん ど移動せずーケ所に止って回転している（図 3－B）。
（iii）直進形運動
更に，整列作用か加わると小さくひとかたまりにな って直進運動をする（図 $3-\mathrm{C}$ ）。

## Dry particles in Japanese article：Sakai model



3
4

$\rightarrow$

Other dry particles
There proposed many models after Vicsek including ...

## Dry particles with vision: Hybrid Projection model

- Pearcea, Millera, Rowlandsa, Turner, PNAS 111, 10422 (2014)
$\underline{v}_{i}^{t+1}=\phi_{p} \underline{\delta}_{i}^{t}+\phi_{a} \widehat{\left\langle\underline{v}_{k}^{t}\right\rangle} n . n .+\phi_{n} \underline{\eta}_{i}^{t}$,

$$
\phi_{p}=0.1, \phi_{a}=0.75
$$

$w / o$ blind angle

with blind angle

$\phi_{p}=0.45, \phi_{a}=0.45$
$\phi_{p}=0.175, \phi_{a}=0.45$





## Major categories of self-propelled particles

## Dry particles

in which hydrodynamics is not important ex. flock of birds

Wet particles = Swimmers in which hydrodynamics plays crucial role ex. school of fish


## How can swimmers swim?

Breaststroke


E-coli bacteria


Swimming = Propulsion without external Force or Torque

$$
\mathbf{u}(\mathbf{r}) \sim|\mathbf{r}|^{-2} \quad \text { cf. driven colloid } \quad \mathbf{u}(\mathbf{r}) \sim|\mathbf{r}|^{-1}
$$

## A detailed swimmer model

- boundary element method: Ishikawa, Sekiya, Imai, Yamaguchi, Biophys. J. 93, 2217 (2007)


FIGURE 1 Shape parameters for a bacterial model with 360 and 320 triangle elements for the flagellum and spherical body, respectively. The total number of elements per bacterium is 680. (a) Shape A $\left(h=0.77, k=k_{\mathrm{E}}=1.3\right.$, and $\left.a_{\mathrm{f}}=0.1\right)$, (b) shape $\mathrm{B}(h=0.77 / 2)$, and $(c)$ shape $\mathrm{C}(h=$ $0.77 / 2, k=2.6$ ).
(a)

(c)

(b)

(d)


## More detailed swimmer model

- PMC dynamics:


## Hu, Yang, Gompper, Winkler, Soft Matter 11, 7867 (2015)


(a)

(c)


Fig. 5 Time-averaged flow field generated by a single swimming bacterium as obtained from simulations: (a) flow field in the swimming plane (b) the theoretical flow pattern for a finite-distance force dipole as illustrated in Fig. 6 (a) as superposition of two Stokeslets within the same periodic box as for our simulations. ( $\mathrm{c}-\mathrm{g}$ ) Flow fields in planes perpendicular to the swimming plane at positions indicated by the white vertical lines in (a). The streamlines indicate the flow direction, and the logarithmic color scheme indicates the magnitude of the flow speed scaled by the bacterial swimming velocity.

## Simpler swimmers in a real-world

Q. Can scallops swim using simple reciprocal motion?
A. Yes, in a real-world (= at moderate Re).
simple reciprocal motion


## Simpler swimmers in a small-scale

A. No, in a small-world (= at low Re).

- Purcell theorem: Purcell, AJP 45, 3 (1977)


Edward Mills Purcell (1912-1997)
1952 Nobel Prize for Physics: Discovery of NMR

Reciprocal
$\langle V\rangle=0$

Non-reciprocal (Circulative) $\langle V\rangle \neq 0$


[^0]
## A model microswimmer (3-linked spheres)

- 3-sphere NG model: Najafi, Golestanian, PRE 69, 062901
(2004)


$$
\begin{aligned}
& v_{1}=\frac{f_{1}}{6 \pi \eta a_{1}}+\frac{f_{2}}{4 \pi \eta L_{1}}+\frac{f_{3}}{4 \pi \eta\left(L_{1}+L_{2}\right)} \\
& v_{2}=\frac{f_{1}}{4 \pi \eta L_{1}}+\frac{f_{2}}{6 \pi \eta a_{2}}+\frac{f_{3}}{4 \pi \eta L_{2}} \\
& v_{3}=\frac{f_{1}}{4 \pi \eta\left(L_{1}+L_{2}\right)}+\frac{f_{2}}{4 \pi \eta L_{2}}+\frac{f_{3}}{6 \pi \eta a_{3}}
\end{aligned}
$$

FIG. 2. Complete cycle of the proposed nonreciprocal motion of the swimmer, which is composed of four consecutive time-reversal breaking stages: (a) the left arm decreases its length with the constant relative velocity $W$, (b) the right arm decreases its length with the same velocity, (c) the left arm opens up to its original length, and finally, (d) the right arm elongates to its original size. By completing the cycle the whole system is displaced to the right side by an amount $\Delta$.

$$
\begin{array}{ll}
L_{1}=l_{1}+u_{1}, & u_{1}(t)=d_{1} \cos \left(\omega t+\varphi_{1}\right) \\
L_{2}=l_{2}+u_{2}, & u_{2}(t)=d_{2} \cos \left(\omega t+\varphi_{2}\right)
\end{array}
$$

$$
\overline{V_{0}}=\frac{K}{2} d_{1} d_{2} \omega \sin \left(\varphi_{1}-\varphi_{2}\right)
$$

## A model microswimmer (2-linked spheres)

- push-me-pull-you (PMPY) model:

Avron, Kenneth, Oaknin, NJP 7, 234 (2005)
Silverberg et al., Bioinspiration and Biomimetics 15, 64001 (2020)


| $a_{1}(t)$ | $a_{2}(t)$ | $l(t)$ |
| :---: | :---: | :---: |
| 1) $a_{S} \rightarrow a_{L}$ | $a_{L} \rightarrow a_{S}$ | $l_{S}$ |
| 2) | $a_{L}$ | $a_{S}$ |$l_{S} \rightarrow l_{L}$

w/o HI (2005)
$\quad X=\frac{a_{L}-a_{S}}{a_{L}+a_{S}}\left(l_{L}-l_{S}\right)$
with HI by Oseen tensor (2020)
3) $a_{L} \rightarrow a_{S}$
$a_{S} \rightarrow a_{L}$
$l_{L}$
4)

$$
a_{L} \quad l_{L} \rightarrow l_{S}
$$

$$
\dot{v}_{1}+\dot{v}_{2}=0
$$

$$
\begin{aligned}
X= & \frac{a_{L}-a_{S}}{a_{L}+a_{S}}\left(l_{L}-l_{S}\right) \\
& +\frac{3 a_{S} a_{L}}{\left(a_{S}-a_{L}\right)} \ln \frac{l_{L}}{l_{S}}
\end{aligned}
$$

## A model microswimmer (3-linked spheres)

- 3-sphere volume exchange (VE) model:

Wang, Hu, Othmer, IMA 155, 185 (2012)

$$
\dot{v}_{1}+\dot{v}_{2}+\dot{v}_{3}=0
$$

(1) $\delta a_{1}=\delta_{1}>0$ while $\delta a_{3} \equiv 0$;
(2) $\delta a_{3}=\delta_{3}>0$ while $\delta a_{1} \equiv 0$;
(3) $\delta a_{1}=-\delta_{1}>0$ while $\delta a_{3} \equiv 0$;
(4) $\delta a_{3}=-\delta_{3}>0$ while $\delta a_{1} \equiv 0$.

## Comparisons of model microswimmers

- NG vs PMPY vs VE
total volume in all spheres are the same, and the Wang, Othmer, MBE 12, 1303 (2015)



## A model microswimmer (3-linked spheres)

- Thermally driven elastic microswimmer: Hosaka, Yasuda, Sou, Okamoto, Komura, JPSJ 86, 113801 (2017)

Oseen tensor

$$
\left(\begin{array}{l}
\dot{x}_{1} \\
\dot{x}_{2} \\
\dot{x}_{3}
\end{array}\right)=\left(\begin{array}{lll}
M_{12} & M_{12} & M_{13} \\
M_{21} & M_{22} & M_{23} \\
M_{31} & M_{32} & M_{33}
\end{array}\right) \cdot\left(\begin{array}{l}
f_{1} \\
f_{2} \\
f_{3}
\end{array}\right)+\left(\begin{array}{l}
\xi_{1} \\
\xi_{2} \\
\xi_{3}
\end{array}\right)
$$

$$
\begin{gathered}
\left\langle\xi_{i}(t) \xi_{j}\left(t^{\prime}\right)\right\rangle=k_{B} \bar{T}_{i j} M_{i j} \delta\left(t-t^{\prime}\right) \\
\bar{T}_{i i}=T_{i}, \quad \bar{T}_{i j}=\operatorname{ave}\left[T_{i}, T_{j}\right]
\end{gathered}
$$

$F_{A}=-K_{A}\left(x_{2}-x_{1}-l\right)$

$$
F_{B}=-\lambda K_{A}\left(x_{3}-x_{2}-l\right)
$$

$$
\begin{aligned}
& f_{1}=-F_{A} \\
& f_{2}=F_{A}-F_{B} \\
& f_{3}=F_{B}
\end{aligned}
$$

$$
\begin{aligned}
\langle V\rangle & \equiv \frac{\left(\dot{x}_{1}+\dot{x}_{3}+\dot{x}_{3}\right)}{3} \\
& =\frac{k_{B}}{144 \pi \eta l^{2}(1+\lambda)}\left[(2-5 \lambda) T_{1}-(7-7 \lambda) T_{2}+(5-2 \lambda) T_{3}\right]
\end{aligned}
$$

Temperature gradient drives the elastic microswimmer

## A model microswimmer (3-linked spheres)

- Thermally driven odd-elastic microswimmer: Yasuda, Hosaka, Sou, Komura, JPSJ 90, 075001 (2021)

Oseen tensor

$$
\binom{F_{A}}{F_{B}}=-\left(\begin{array}{cc}
K^{e} & K^{0} \\
-K^{0} & K^{e}
\end{array}\right) \cdot\binom{x_{2}-x_{1}-l}{x_{3}-x_{2}-l}
$$

$$
\begin{aligned}
& f_{1}=-F_{A} \\
& f_{2}=F_{A}-F_{B} \\
& f_{3}=F_{B}
\end{aligned}
$$

$$
\begin{gathered}
\left(\begin{array}{c}
\dot{x}_{1} \\
\dot{x}_{2} \\
\dot{x}_{3}
\end{array}\right)=\left(\begin{array}{lll}
M_{12} & M_{12} & M_{13} \\
M_{21} & M_{22} & M_{23} \\
M_{31} & M_{32} & M_{33}
\end{array}\right) \cdot\left(\begin{array}{l}
f_{1} \\
f_{2} \\
f_{3}
\end{array}\right)+\left(\begin{array}{l}
\xi_{1} \\
\xi_{2} \\
\xi_{3}
\end{array}\right) \\
\left\langle\xi_{i}(t) \xi_{j}\left(t^{\prime}\right)\right\rangle=k_{B} T M_{i j} \delta\left(t-t^{\prime}\right)
\end{gathered}
$$

$$
\langle V\rangle \equiv \frac{\left(\dot{x}_{1}+\dot{x}_{3}+\dot{x}_{3}\right)}{3}=\frac{7 a k_{B} T \lambda}{8 l^{2} K^{e} \tau}+\mathcal{O}
$$

$$
\begin{aligned}
& \lambda \equiv K^{0} / K^{e} \\
& \tau \equiv 6 \pi \eta a / K^{e}
\end{aligned}
$$

The odd-elastic microswimmer can spontaneously propel in a heat bath!!

## A model microswimmer (3-linked rods)

- Purcell's swimmer with odd-elasticity: Ishimoto, Moreau, Yasuda, PRE 105, 064603 (2022)

$\binom{T_{1}}{T_{2}}=-\left(\begin{array}{cc}\kappa_{e} & \kappa_{0} \\ -\kappa_{0} & \kappa_{e}\end{array}\right) \cdot\binom{\alpha_{1}}{\alpha_{2}} \quad \gamma \equiv \kappa_{0} / \kappa_{e}$
Spontaneous and autonomous motion can be sustained in viscous media by the odd-elasticity!!! 27


# Unsolved problems of microswimmers 

(of my personal choice)

## A spherical microswimmer

- Squirmer model: Lighthill (1952), Blake (1971)

Propelling axis


Sliding velocity $\boldsymbol{u}^{(s)}$ using polynomial expansion

$$
\begin{aligned}
& \boldsymbol{u}^{(s)}=\sum_{n=1}^{n=\infty} \frac{2}{n(n+1)} B_{n}(\hat{\boldsymbol{e}} \cdot \hat{\boldsymbol{r}} \hat{\boldsymbol{r}}-\hat{\boldsymbol{e}}) P_{n}^{\prime}(\hat{\boldsymbol{e}} \cdot \hat{\boldsymbol{r}}) \\
&=\sum_{n=1}^{\infty} \frac{2}{n(n+1)} B_{n} P_{n}^{\prime}(\cos \theta) \sin \theta \hat{\boldsymbol{\theta}} \\
& \downarrow \text { neglecting } \mathrm{n}>2
\end{aligned} \mathbf{u}^{(s)}=\left(B_{1} \sin \theta+B_{2} \sin 2 \theta\right) \hat{\boldsymbol{\theta}} \quad \$
$$

## A spherical microswimmer



## Single squirmer motions

Pusher
$\alpha=-2$
Neutral

$$
\alpha=0
$$

> Puller
> $\alpha=2$

$$
\alpha \equiv \frac{B_{2}}{B_{1}}
$$



$$
\boldsymbol{U}=\frac{2}{3} B_{1} \hat{\boldsymbol{e}}
$$

$$
\mathbf{u}(\mathbf{r}) \sim|\mathbf{r}|^{-2}
$$

$$
\mathbf{u}(\mathbf{r}) \sim|\mathbf{r}|^{-3}
$$

$$
\mathbf{u}(\mathbf{r}) \sim|\mathbf{r}|^{-2}
$$

Box: $64 \times 64 \times 64$ with PBC, Particle radius: $a=6, \phi=0.002$

## Collective behaviors of many pushers /pullers

Confined in parallel walls (V.F. of Squirmer $=0.13$ )


## Collective behaviors of many pushers /pullers



## Collective behaviors of many pushers /pullers



Efficient propulsion in viscoelastic media at low Re

## Squirmer in viscoelastic fluid

## Oldroyd-B model

total stress:

$$
\begin{aligned}
& \mathbf{T}=2 \eta_{s} \mathbf{D}+\tau \\
& \quad \text { solvent polymer } \\
& \tau+\lambda_{1} \stackrel{\nabla}{\tau}=2 \eta_{p} \mathbf{D}
\end{aligned}
$$

where

$$
\begin{array}{r}
\beta \equiv \frac{\eta_{s}}{\eta_{s}+\eta_{p}} \\
\mathrm{Wi} \equiv \frac{\lambda_{1} B_{1}}{a}
\end{array}
$$

upper convective Maxwell model
with

$$
\begin{aligned}
& \mathbf{\nabla}=\frac{\partial}{\partial t} \mathbf{T}+\mathbf{v} \cdot \nabla \mathbf{T}-\left((\nabla \mathbf{v})^{T} \cdot \mathbf{T}+\mathbf{T} \cdot(\nabla \mathbf{v})\right) \\
& \mathbf{D}=\frac{1}{2}\left[\nabla \mathbf{v}+(\boldsymbol{\nabla} \mathbf{v})^{T}\right]
\end{aligned}
$$

## Squirmer in viscoelastic fluid


T. Kobayashi (on-going)

## Squirmer in viscoelastic fluid

## Squirmer with only rotlet dipole



Neutral squirmer with the rotlet dipole

T. Kobayashi (in future)

## Efficient propulsion in viscoelastic media



Weissenberg effect
Visco-elastic propeller

Thank you for your kind attention.

## Model microswimmer (1 sphere)

## - DDFT model:

Menzel, Saha, Hoell, Löwen, JCP 144, 024115 (2016)


FIG. 1. Individual model microswimmer. The spherical swimmer body of hydrodynamic radius $a$ is subjected to hydrodynamic drag. Two active pointlike force centers exert active forces $+\mathbf{f}$ and $-\mathbf{f}$ onto the surrounding fluid. This results in a self-induced fluid flow indicated by small light arrows. $L$ is the distance between the two force centers. The whole setup is axially symmetric with respect to the axis $\hat{\mathbf{n}}$. If the swimmer body is shifted along $\hat{\mathbf{n}}$ out of the geometric center, leading to distances $\alpha L$ and $(1-\alpha) L$ to the two force centers, it feels a net self-induced hydrodynamic drag. The microswimmer then self-propels. In the depicted state (pusher), fluid is pushed outward. Upon inversion of the two forces, fluid is pulled inward (puller). We consider soft isotropic steric interactions between the swimmer bodies of typical interaction range $\sigma$, implying an effective steric swimmer radius of $\sigma / 2$.
$x_{\text {Puller }}$

## Swarm propagation: Mixtures


0.6

0.7


0.5
 ,

0.8

$x_{\text {Puller }}$

## Swarm propagation: Mixtures


0.1


## 0.0




## Collective motions in bulk

Pusher
$\alpha=-2$


Neutral
$\alpha=0$


Puller
$\alpha=2$

## Spherical micro-swimmer with rotlet



$B_{1} \sin \theta \widehat{\mathbf{e}}_{\theta}$

$$
B_{2} \sin 2 \theta \widehat{\mathbf{e}}_{\theta}
$$

$$
\chi \equiv \frac{C_{2}}{B_{1}}
$$


$C_{2} \sin 2 \theta \hat{\mathbf{e}}_{\varphi}$ azimuthal


[^0]:    Purcell's swimmer

