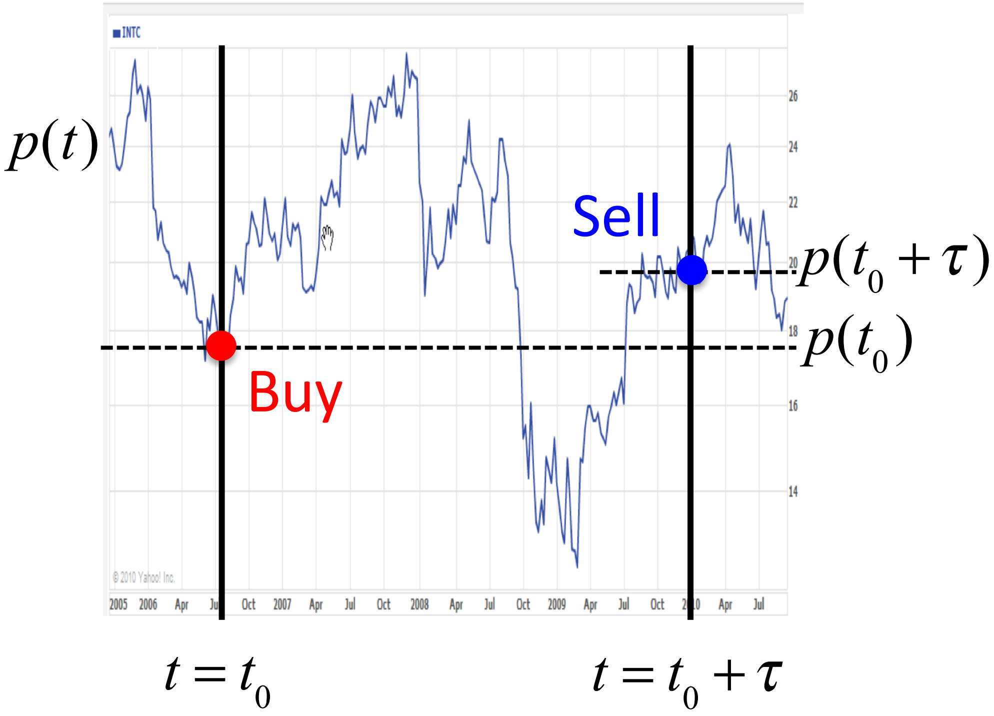
Lab1 Seminar
2018/07/30

Microscopic stochastic dealer models for financial markets

Recent papers of Takayasu et al.

Ryoichi Yamamoto

Financial market



Return: $R_\tau = p(t_0+\tau) - p(t_0)$

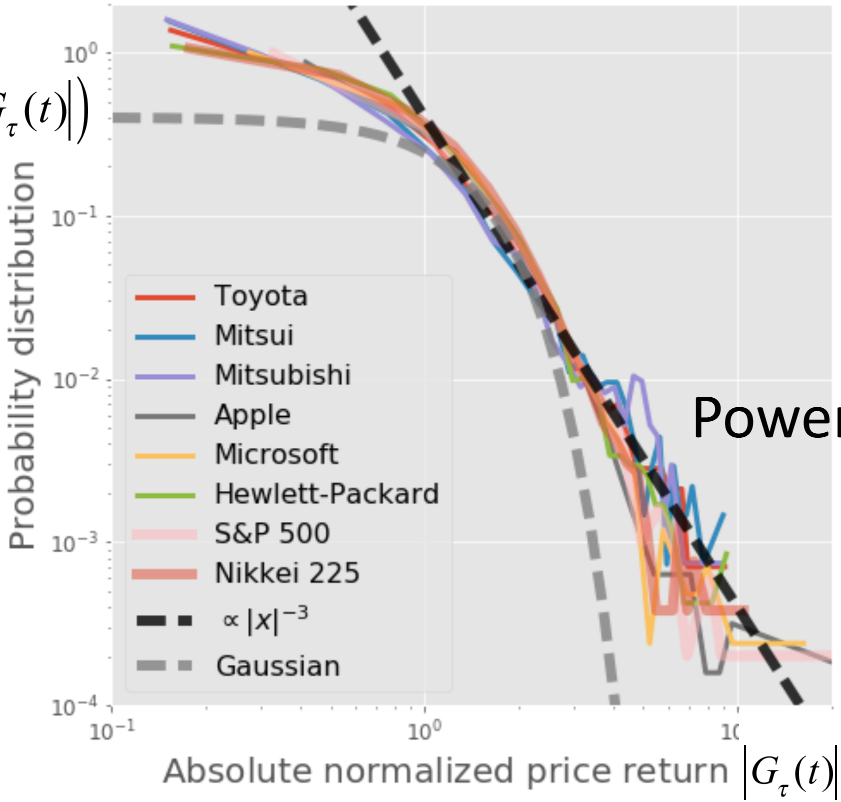
Log return: $G_\tau = \ln \dfrac{p(t_0+\tau)}{p(t_0)}$

Return rate: $r_\tau = \dfrac{p(t_0+\tau) - p(t_0)}{p(t_0)}$

$$G_\tau = r_\tau \quad \text{for} \quad \tau \to 0$$

Unsolved problem

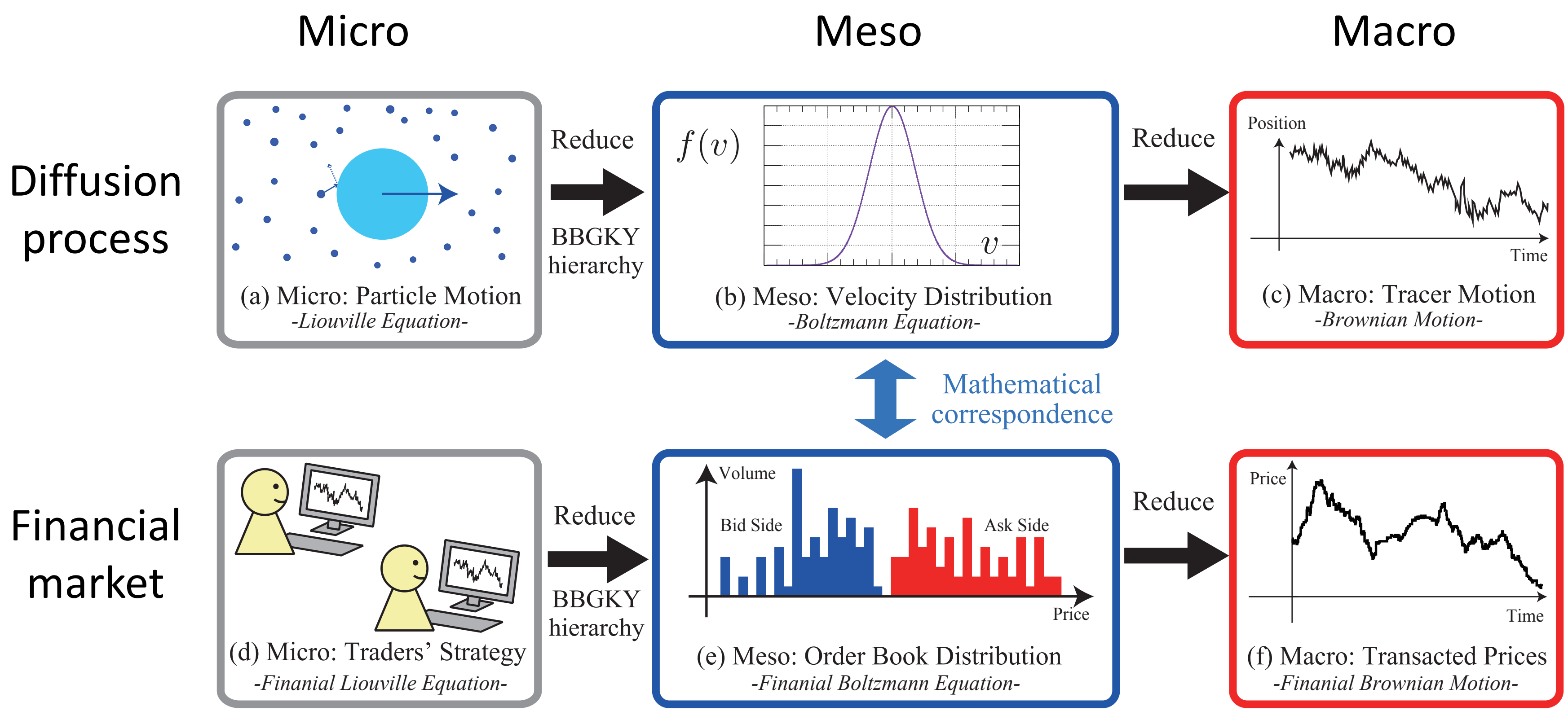
$P\left(\left|G_\tau(t)\right|\right)$



Power law !?

$\left|G_\tau(t)\right| = \left|\ln \frac{p(t+\tau)}{p(t)}\right|$

Diffusion vs. finance



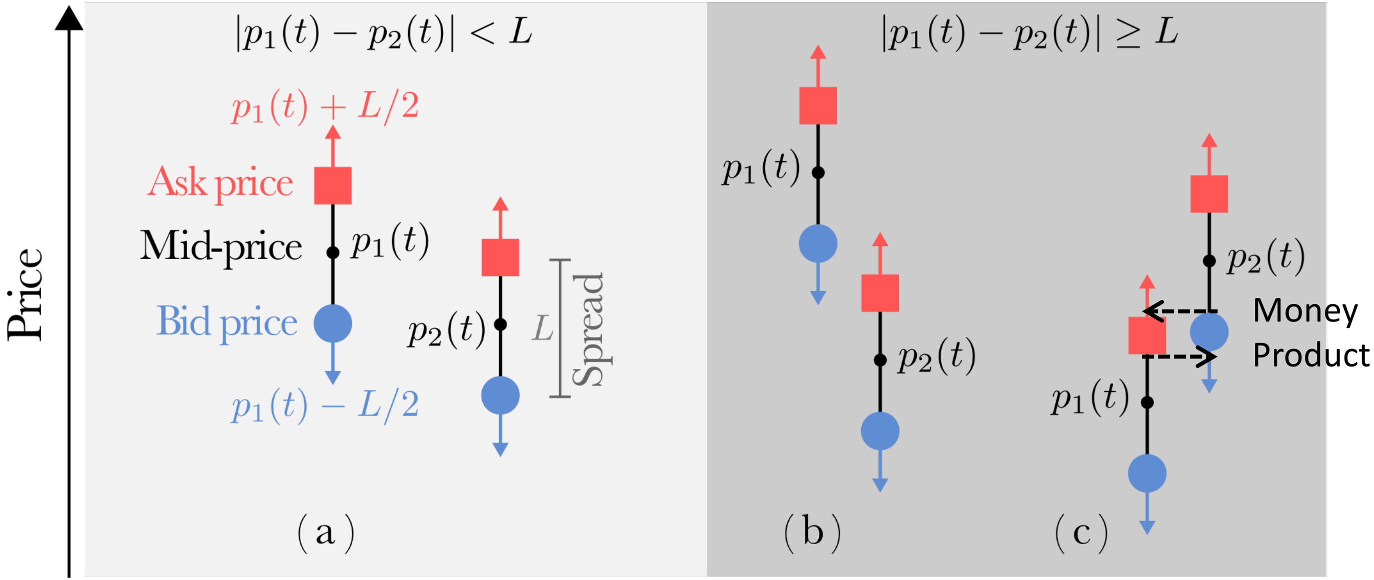
Microscopic models

1. K. Yamada, H. Takayasu, T. Ito, & M. Takayasu, Solvable stochastic dealer models for financial markets. *Physical Review* **79**, 051120 (2009).
   - Model 1: 2-body stochastic dealer model
   - Model 2: 2-body stochastic dealer model + transaction interval
   - Model 3: 2-body stochastic dealer model + trend-following

2. K. Kanazawa, T. Sueshige, H. Takayasu & M. Takayasu, Derivation of the Boltzmann Equation for Financial Brownian Motion: Direct Observation of the Collective Motion of High-Frequency Traders. *Phys. Rev. Lett.* **120,** 138301 (2018).
   - Model 4: N-body stochastic dealer model + trend-following

Model 1: 2-body stochastic dealer model

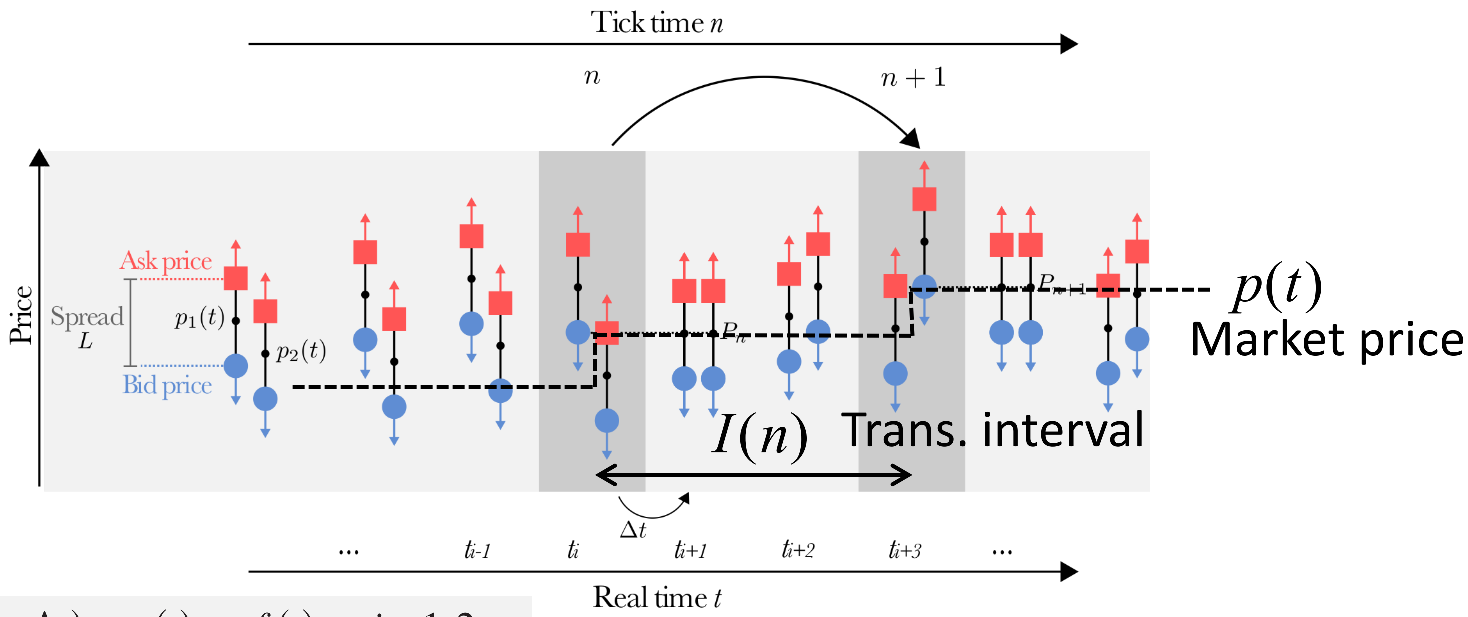
No transaction (Ask > Bid) Transaction (Ask ≦ Bid)



Ask price: a minimum price he is willing to accept to sell a product

Bid price: a maximum price he is willing to pay to buy a product

Model 1: 2-body stochastic dealer model



$$p_i(t+\Delta t) = p_i(t) + c f_i(t), \quad i = 1,2,$$

$$f_i(t) = \begin{cases} +\Delta p & (\text{prob. } 1/2) \\ -\Delta p & (\text{prob. } 1/2). \end{cases}$$

No transaction

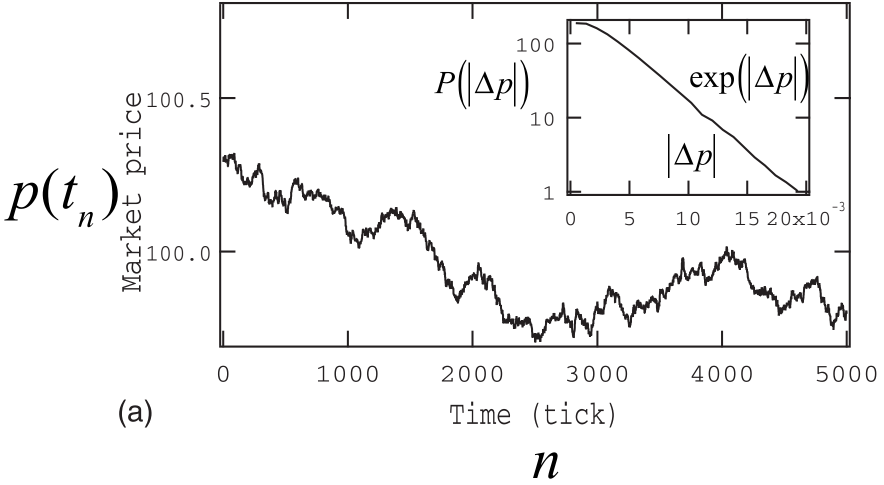
$$p(t+0) = a_2(t) = b_1(t)$$

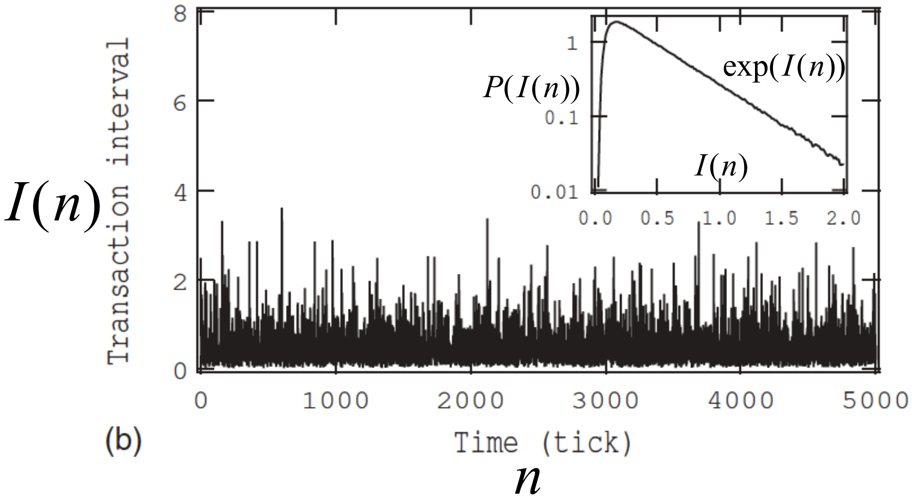
$$\Delta p(t+0) = p(t+0) - p(t-0)$$

$$p_1(t+0) = p_2(t+0) = p(t+0)$$

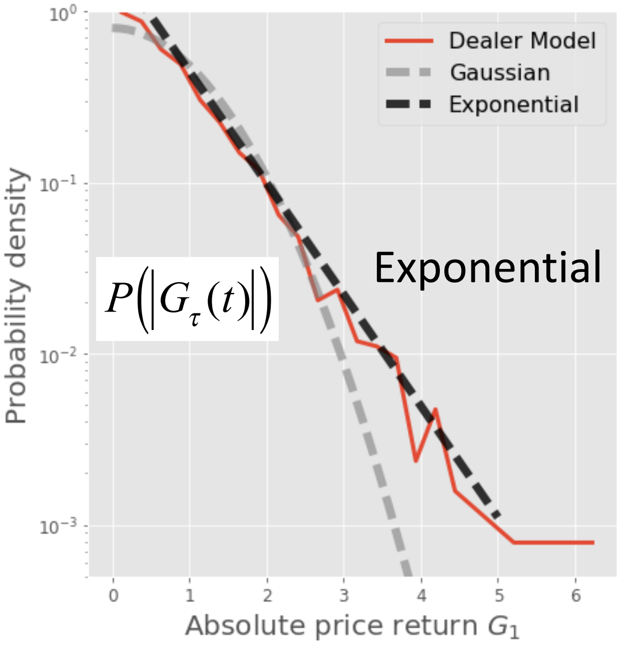
Transaction

Model 1: 2-body stochastic dealer model

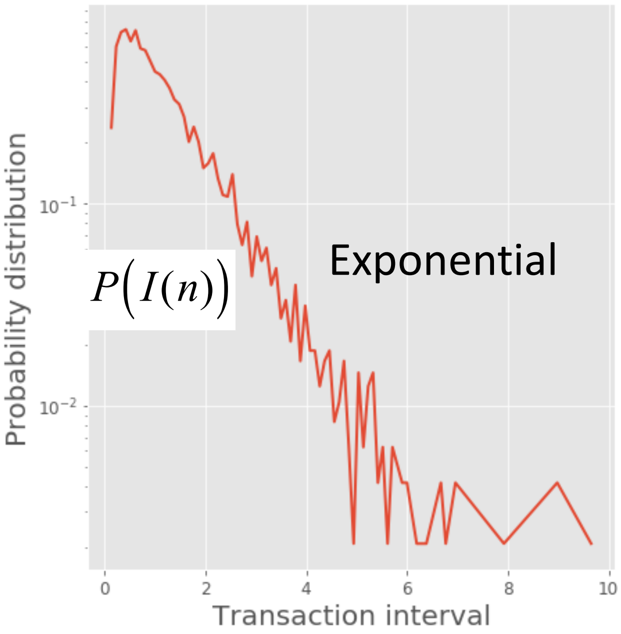




Model 1: 2-body stochastic dealer model



$$|G_\tau(t)| = \left| \ln \frac{p(t+\tau)}{p(t)} \right|$$

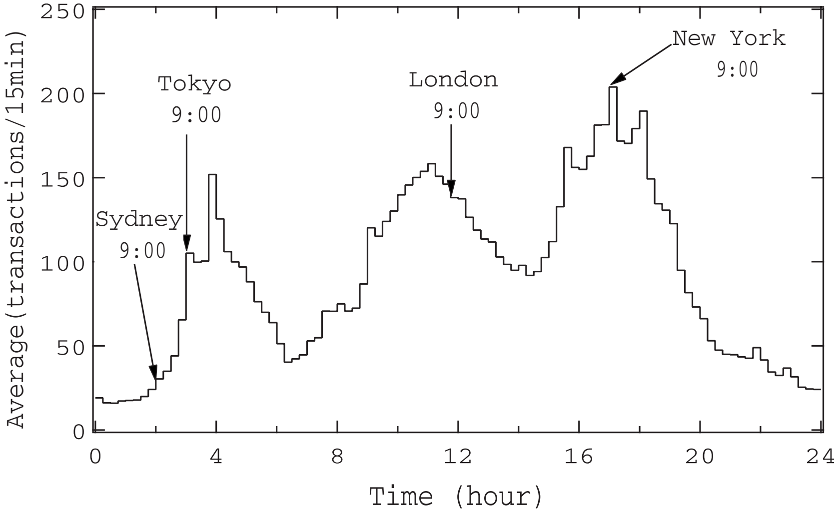


$$I(n)$$

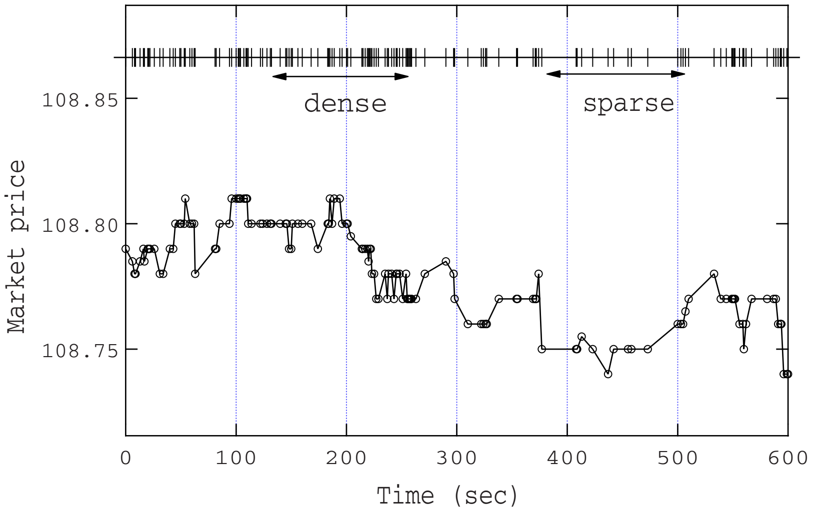
Poisson process

Model 2: 2-body stochastic dealer model + transaction interval

long time behavior in FX



short time behavior in FX



Apparent non-Poisson process

Model 2: 2-body stochastic dealer model + transaction interval

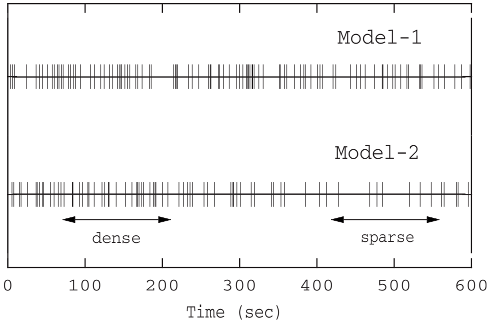
$$p_i(t+\Delta t) = p_i(t) + c(n) f_i(t), \quad i = 1, 2,$$

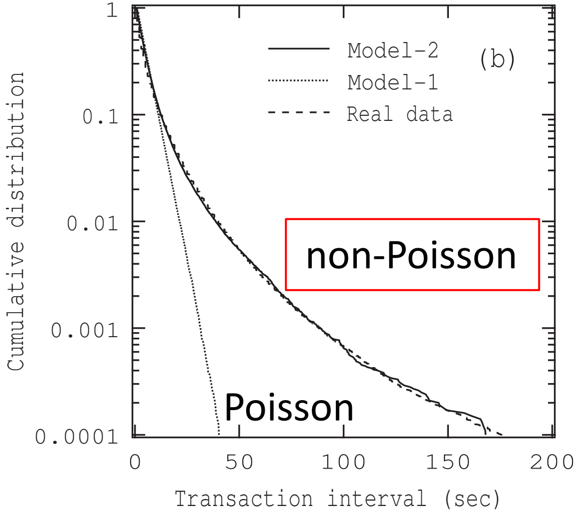
$$f_i(t) = \begin{cases} +\Delta p & (\text{prob. } 1/2) \\ -\Delta p & (\text{prob. } 1/2), \end{cases}$$

where

$$c(n) = \sqrt{\frac{\langle I \rangle_{c=1}}{\langle I \rangle_\tau}}.$$

$$\langle I \rangle_\tau = \frac{1}{N} \sum_{k=0}^{N-1} I(n-k)$$





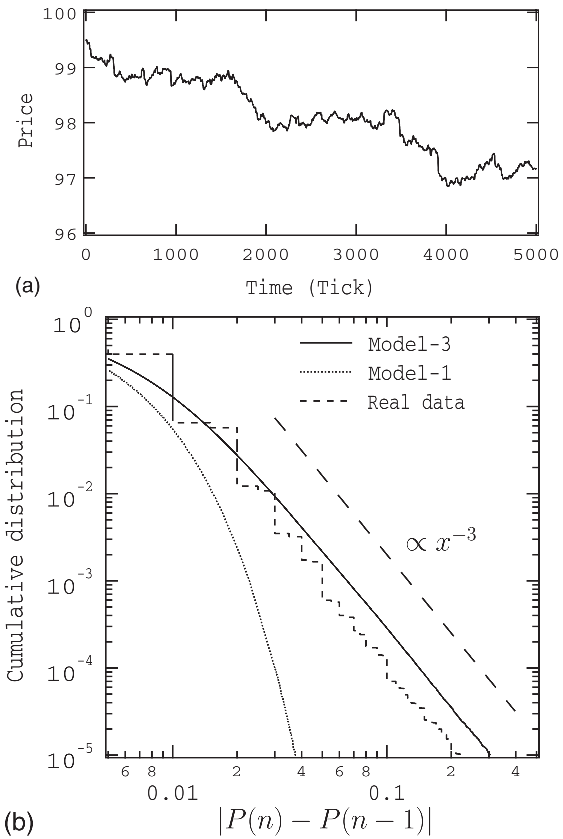
Model 3: 2-body stochastic dealer model + trend-following

$$p_i(t+\Delta t) = p_i(t) + \underline{d\langle \Delta P\rangle_M \Delta t} + c f_i(t),$$

$$f_i(t) = \begin{cases} +\Delta p & (\text{prob. } 1/2) \\ -\Delta p & (\text{prob. } 1/2) \end{cases} \qquad i = 1, 2,$$

where

$$\langle \Delta P\rangle_M = \frac{2}{M(M+1)} \sum_{k=0}^{M-1} (M-k)\Delta P(n-k).$$

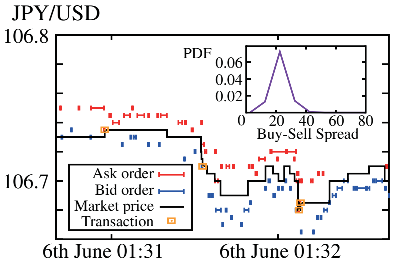


Power law !!

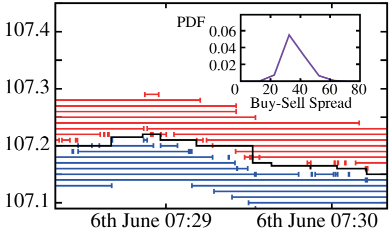
Model 4: N-body stochastic dealer model + trend-following

1. Observed microscopic dynamics (big data...)

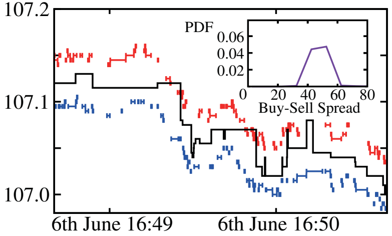
HFTs
in FX



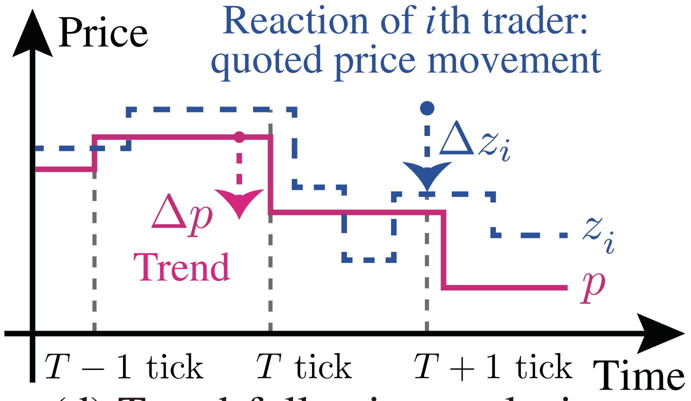
(a) 1st top HFT



(b) 2nd top HFT

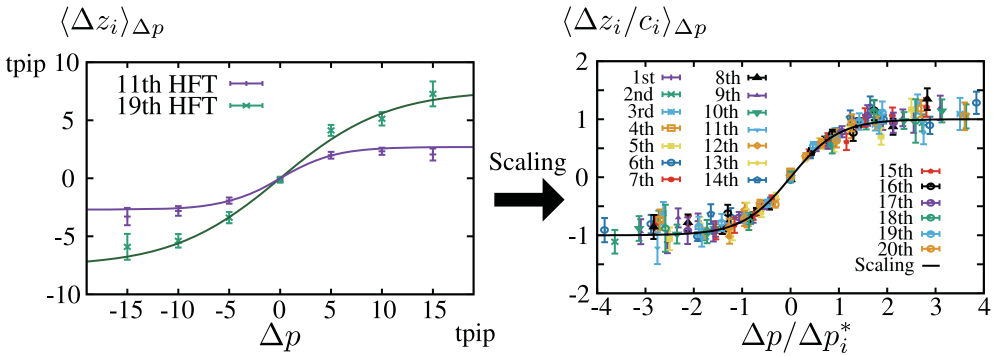


(c) 3rd top HFT



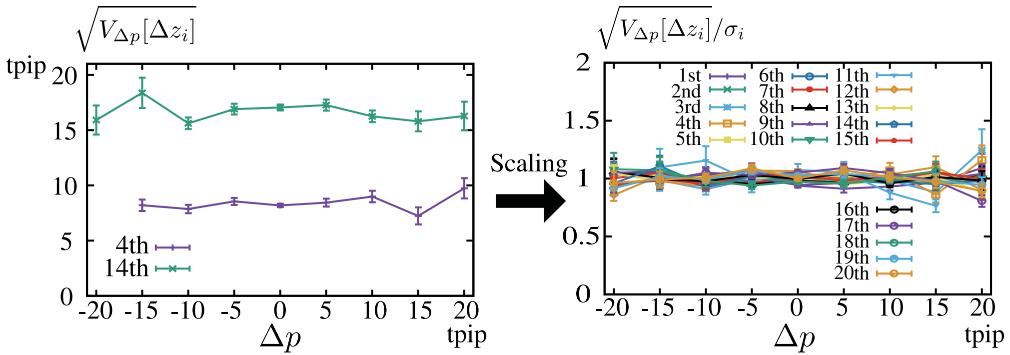
(d) Trend-following analysis

Model 4: N-body stochastic dealer model + trend-following



(a) Trend-following movement on average (hyperbolic function)

$$\langle \Delta z_i \rangle_{\Delta p} \approx c_i \tanh \frac{\Delta p}{\Delta p_i^*}$$



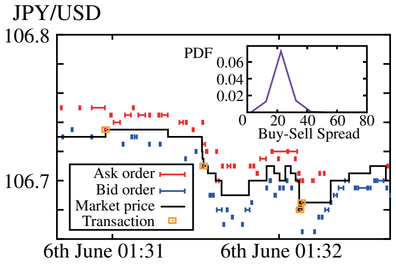
(b) Standard deviation of random noise effect (independent of Δp)

$$V_{\Delta p}[\Delta z_i] \approx \sigma_i^2$$ (constant)

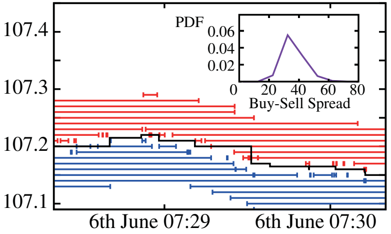
Characteristic constants unique to the *i*-th dealer" $c_i, \Delta p_i^*, \sigma_i^2$

Model 4: N-body stochastic dealer model + trend-following

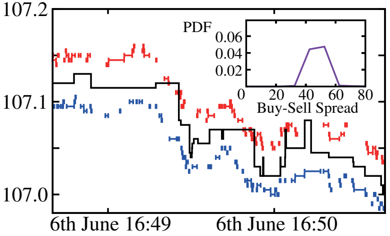
Individual traders (micro)



(a) 1st top HFT

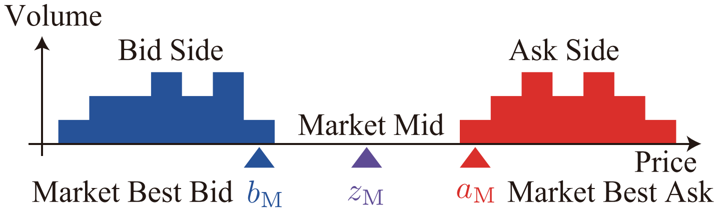


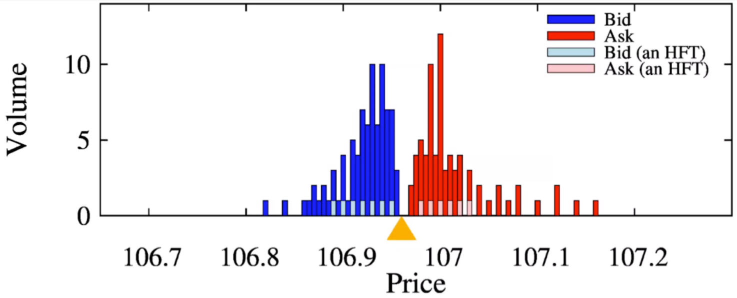
(b) 2nd top HFT



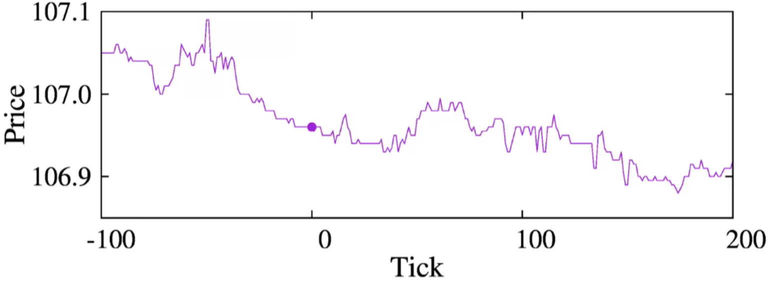
(c) 3rd top HFT

Order-book (meso)



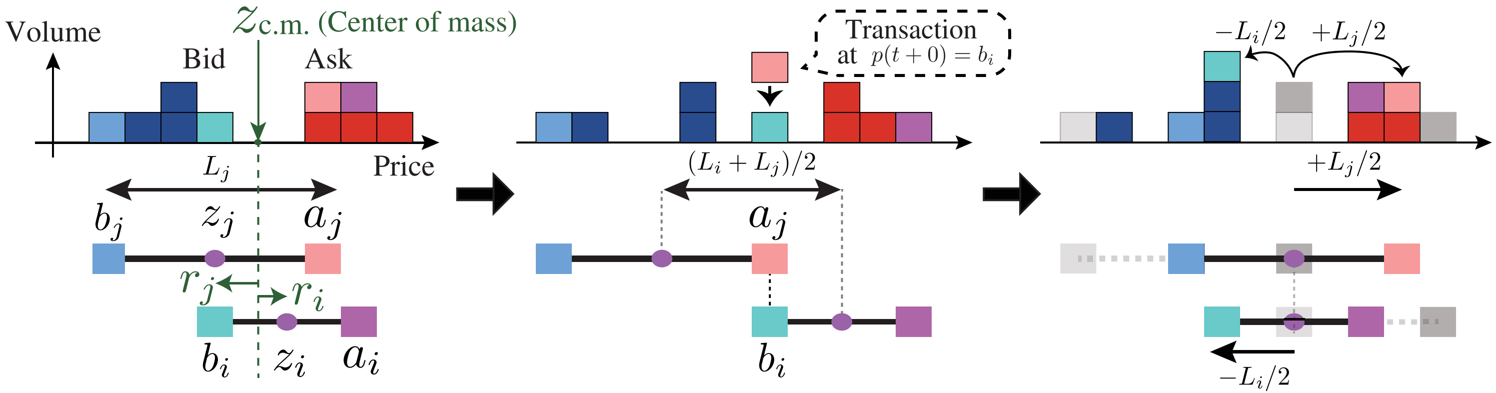


Market price (macro)



Model 4: N-body stochastic dealer model + trend-following

2. Microscopic model



(a) Trend-following random walks (b) Transaction (c) Requotation jump

$$\frac{dz_i(t)}{dt} = c \tanh \frac{\Delta p(t)}{\Delta p^*} + \sigma \eta_i^R(t)$$

$$p(t+0) = b_i(t)$$

$$\Delta p(t+0) = b_i(t) - p(t)$$

$$z_i(t+0) = z_i(t) - \frac{L_i}{2}$$

$$z_j(t+0) = z_j(t) + \frac{L_j}{2}$$

cf. Model 3

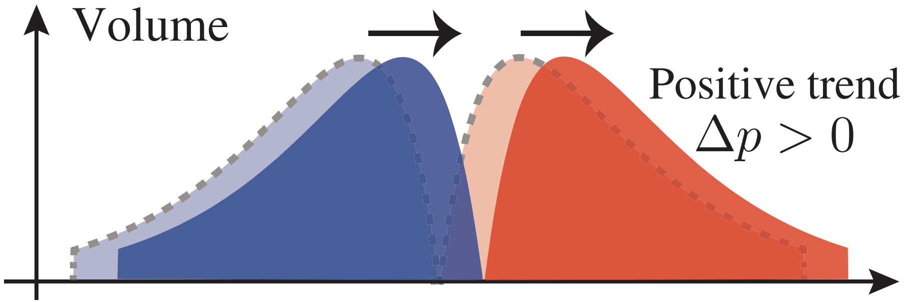
$$p_i(t+\Delta t) = p_i(t) + d\langle \Delta P \rangle_M \Delta t + c f_i(t),$$

Model 4: N-body stochastic dealer model + trend-following

2. Microscopic model

Trend following R.W. ⟶ Corrective motion in order book

$$\frac{dz_i(t)}{dt} = c \tanh\frac{\Delta p(t)}{\Delta p^*} + \sigma\eta_i^R(t)$$



(d) Collective motion of order book

Self organization
(mean field)

Model 4: N-body stochastic dealer model + trend-following

3. Kinetic formulation

Trend following R.W.
(in single Eq.)

$$\frac{dz_i}{dt} = c\tanh\frac{\Delta p}{\Delta p^*} + \sigma\eta_i^{\mathrm{R}} + \eta_i^{\mathrm{T}}$$

where

$$\eta_i^{\mathrm{T}} \equiv \sum_{k=1}^{\infty}\sum_{j}^{j\neq i}\Delta z_{ij}\delta(t-\tau_{k;ij})$$

$$\Delta z_{ij} = -\frac{L_i}{2}\mathrm{sgn}(z_i - z_j)$$

Define new variables

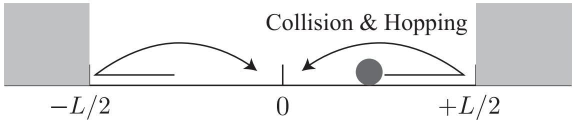
$$z_{\mathrm{c.m.}} \equiv \frac{1}{N}\sum_{i=1}^{N} z_i$$

$$\frac{dz_{\mathrm{c.m.}}}{dt} = c\tanh\frac{\Delta p}{\Delta p^*} + \xi$$

$$\xi \equiv \frac{1}{N}\sum_{j=1}^{N}\left(\sigma\eta_j^{\mathrm{R}} + \eta_j^{\mathrm{T}}\right)$$

$$r_i \equiv z_i - z_{\mathrm{c.m.}}$$

$$\frac{dr_i}{dt} = \sigma\eta_i^{\mathrm{R}} + \eta_i^{\mathrm{T}} - \xi \approx \sigma\eta_i^{\mathrm{R}} + \eta_i^{\mathrm{T}}$$



Model 4: N-body stochastic dealer model + trend-following

3. Kinetic formulation

1-body PDF in N-body system

$$\frac{\partial \phi_L(r)}{\partial t} = \frac{\sigma^2}{2}\frac{\partial^2 \phi_L(r)}{\partial r^2} + C(\phi_{LL'})$$

Diffusion   Collision

$$C(\phi_{LL'}) = N \sum_{s=\pm 1} \int dL' \rho(L') \left[J^s_{LL'}(r + sL/2) - J^s_{LL'}(r)\right]$$

$$J^s_{LL'}(r) = \frac{\sigma^{*2}}{2} |\tilde{\partial}_{rr'}| \phi_{LL'}(r, r') \Big|_{r-r'=s(L+L')/2}$$

2-body PDF

(BBGKY hierarchy)

Decomposition of 2-boby PDF

$$\phi_{LL'}(r, r') \approx \phi_L(r)\phi_{L'}(r')$$

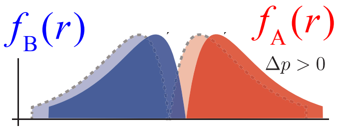
↓

Boltzmann-like Eq.

↓

$$\psi_L(r) \equiv \lim_{t\to\infty} \lim_{N\to 0} \phi_L(r;t) = \frac{4}{L^2} \max\left\{\frac{L}{2} - |r|, 0\right\}$$

$$f_{\mathrm{A}}(r) = \int dL \rho(L) \psi_L(r - L/2)$$



Model 4: N-body stochastic dealer model + trend-following

4. Mesoscopic and macroscopic data analysis

Input
(empirical func.)

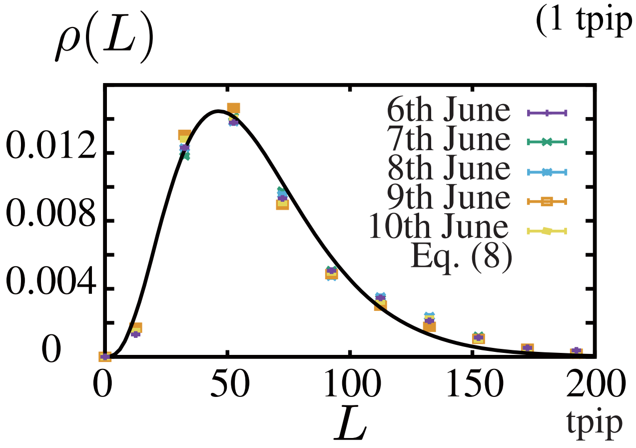
$$\rho(L) = \frac{L^3 e^{-L/L^*}}{6L^{*4}}$$

$$L^* = 15.5 \pm 0.2 \text{ tpip}$$

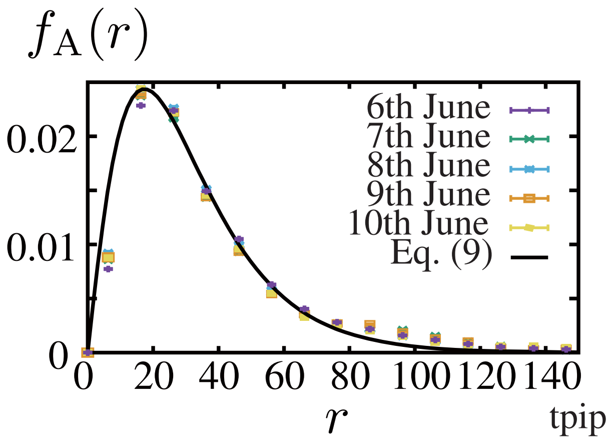
Boltzmann Eq. →

$$f_A(r) = \int dL \rho(L) \psi_L(r - L/2)$$

(1 tpip = ¥ 0.001)



(a) Daily buy-sell spread dist.



(b) Daily order-book profile

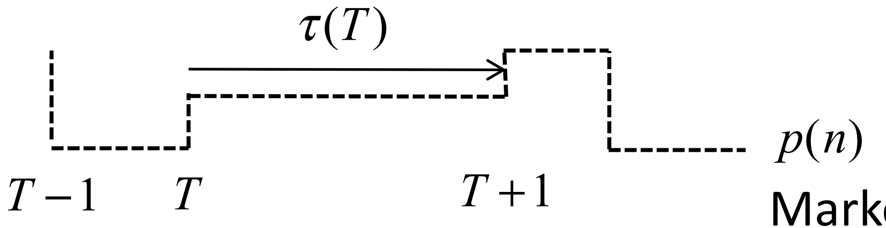
Model 4: N-body stochastic dealer model + trend-following

3. Kinetic formulation

Trend following R.W.

$$\frac{dz_{\mathrm{c.m.}}}{dt} = c\tanh\frac{\Delta p}{\Delta p^*} + \xi$$

$$\Delta p(T+1) = c\tau(T)\tanh\frac{\Delta p(T)}{\Delta p^*} + \zeta(T),$$



Market price

Cumulative PDF

$$P(\geq |\Delta p|;\kappa) \approx e^{-|\Delta p|/\kappa} \quad (|\Delta p| \to \infty)$$

$$\kappa \approx 2\Delta z^*/3$$

$$\Delta z^* \equiv c\tau^*$$

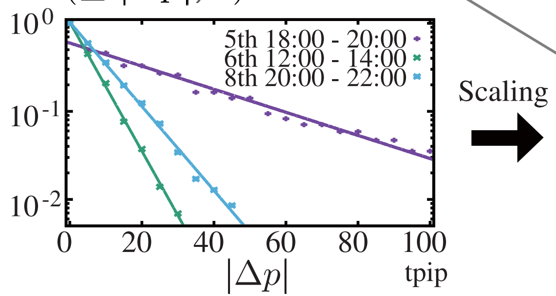
$$\tau^* \approx 3L^{*2}/N\sigma^2$$

$$L^* = 15.5 \pm 0.2 \text{ tpip}$$

Model 4: N-body stochastic dealer model + trend-following

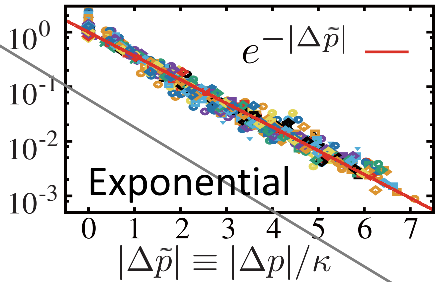
4. Mesoscopic and macroscopic data analysis

$P^{2\mathrm{h}}(\geq |\Delta p|;\kappa) \approx e^{-|\Delta p|/\kappa}$



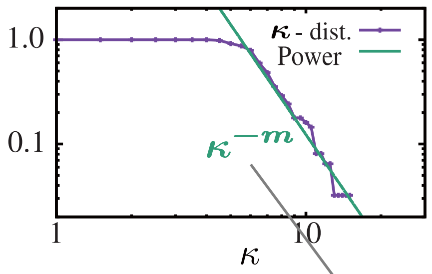
(c) Price movement CDF (one tick)

$\tilde{P}^{2\mathrm{h}}(\geq |\Delta \tilde{p}|)$



(d) Scaled price movement CDF

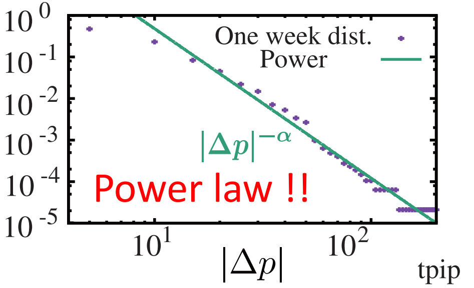
$Q(\geq \kappa)$



(f) Decay length CDF

$Q(\kappa) \equiv -dQ(\geq \kappa)/d\kappa$

$P^{\mathrm{w}}(\geq |\Delta p|)$



(e) One week price movement CDF

$$P^{\mathrm{w}}(\geq |\Delta p|) = \int_0^\infty d\kappa Q(\kappa) P^{2\mathrm{h}}(\geq |\Delta p|;\kappa) \propto |\Delta p|^{-m}$$

$$\alpha \approx m \approx 3.5$$

Model 4: N-body stochastic dealer model + trend-following

5. Summary

1. A microscopic model is developed for FX traders by direct observation of the HFTs' dynamics.
2. A kinetic theory is constructed to show consistencies of the microscopic model with mesoscopic and macroscopic findings.
3. The present model is the first microscopic model directly supported by microscopic dynamical evidence and exhibiting agreement with mesoscopic and macroscopic findings.
4. Introduction of collective motion to order-book models is the key to replicate empirical findings.

Future problems

- Validity check for the kinetic theory
- Use of more data, new data, bigger data
- Diversity (poly-dispersity) in traders / financial instruments
- Beyond mean field, non-global coupling
- Memory, fictitious mass
- Very fast dynamics
- Mechanisms of financial crash
- ...

Performing computer simulations is promising!!

Other related references

1. R. Yamamoto & John J. Molina, “Stochastic Processes: Data Analysis and Computer Simulation” *Kyoto Ux-009x (edX)* (2017)
2. K. Kanazawa, T. Sueshige, H. Takayasu, and M. Takayasu, Kinetic Theory for Finance Brownian Motion from Microscopic Dynamics, *arXiv:1802.05993*.
3. Y. Yura, H. Takayasu, D. Sornette, & M. Takayasu, Financial Knudsen number: Breakdown of continuous price dynamics and asymmetric buy-and-sell structures confirmed by high-precision order-book information. *Phys. Rev. E* **92,** 042811 (2015).
4. Y. Yura, H. Takayasu, D. Sornette, & M. Takayasu, Financial Brownian Particle in the Layered Order-Book Fluid and Fluctuation-Dissipation Relations. *Phys. Rev. Lett.* **112,** 098703 (2014).

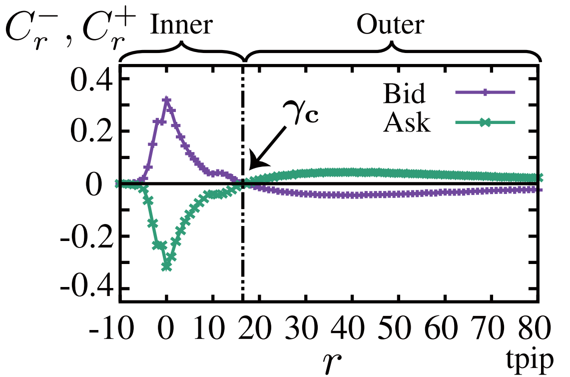
Model 4: N-body stochastic dealer model + trend-following

0. Details of the market

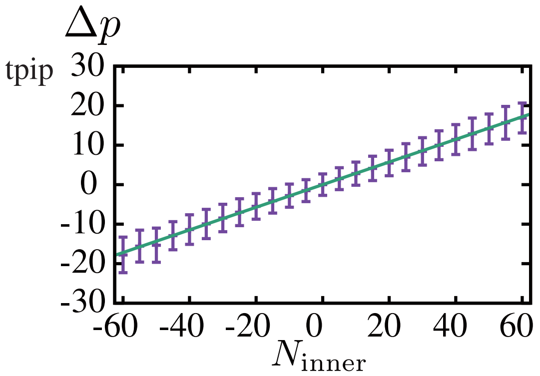
- Market name: Electronic Broking Services (EBS)
- Actions of traders:
  - Limit order: Quoting price with a certain volume and the quoted price displayed on the order book. 922 traders submitted this in the data set.
  - Market order: Buying or selling currencies immediately at the available best price. 93 traders submitted this in the data set.
  - Cancellation
- High frequency trader (HFT):
  - Submit more than 500 times a day on average (134/1015).
  - Rapidly grown recently (87.8% of the total orders were submitted by the HFTs in the data set).

Model 4: N-body stochastic dealer model + trend-following

4. Mesoscopic and macroscopic data analysis



(g) Layered structure of order book



(h) Correlation between N_{inner} & Δp